

Multiplying by One

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If you've ever done the motocross thing, then you've heard the saying "When in doubt, give it gas." Oddly enough, this is actually good advice because extra gas tends to lift the front wheel off the ground, and it's the *front wheel* slamming into things that tends to toss us into the dirt.

Similarly, in math it might be said, "When in doubt, multiply by one." Yes, this might seem trivial on the surface because when you multiply by one, nothing has changed. But when you truly dissect the implication of most math, that's all a lot of it boils down to... multiplying by one.

Ok, just to make sure we're all "on the same page," a few simple questions. What is one times one? One. What is one divided by one? One. What is X divided by X? One. What is Z divided by Z? One. If Z equals X, what is Z divided by X? One.

Wow! This is easy!

Ok, if one mile is 5280 feet, then what is one mile divided by 5280 feet? This isn't quite as easy, but the answer is... one. Let's look at this a bit closer.

$$1.1 \quad \boxed{1 \text{ mile} = 5280 \text{ ft}}$$

If we perform identical processes on both sides of this equation, then the equation remains valid. For example if we divide both sides by 5280 ft, then we retain our equality:

$$1.2 \quad \boxed{\frac{(1 \text{ mile})}{(5280 \text{ feet})} = \frac{(5280 \text{ feet})}{(5280 \text{ feet})}}$$

Because anything divided by itself is 1, we can rewrite this as

$$1.2a \quad \boxed{\frac{(1 \text{ mile})}{(5280 \text{ feet})} = 1}$$

This means that if we multiply something by (1 mile)/(5280 ft), we're actually multiplying by one. This may seem strange, but so it goes.

Similarly, one kilometer = 1000 meters, so (1 km)/(1 m) = 1. One pound = 16 ounces, so (1 lb)/(16 oz.) = 1. Twelve inches = 1 foot, so (12 inches)/(1 foot) = 1.

So where is all of this leading? Suppose something is 14,000 feet long. How many miles (long) is it? Strange as it may seem, if we multiply 14,000 feet by one, we'll have the answer:

$$1.3 \quad (14,000 \text{ feet}) \frac{(1 \text{ mile})}{(5280 \text{ feet})} = 2.65 \text{ mile}$$

As you can see, multiplying by one is very handy. (A reminder about the standard algebraic nomenclature: when two quantities sit beside each other without an intervening symbol such as “+” or “−”, it is assumed that the two quantities are multiplied. So, in equation 1.3, the (14,000 feet) is multiplied by the rest of the stuff on the left-hand side of the equation.)

Now lets look into the details of equation 1.3. We can treat our units (like “ft” and “mile”) like any other symbol. So we can rearrange the left side of equation 1.3 as:

$$1.3a \quad \left(\frac{14,000}{5,280} \right) \left(\frac{\text{feet}}{\text{feet}} \right) \text{ mile}$$

Because (feet/feet) is 1, and because we can eliminate one's from multiplications (Z multiplied by one is still Z), this becomes:

$$1.3b \quad \left(\frac{14,000}{5,280} \right) \text{ mile}$$

Completing the division leaves us with 2.65 mile, as in equation 1.3. So 14,000 feet is 2.65 mile. (Note that you're free to interchange the singular and plural form of “mile”.)

Wow! This multiplying by one is handy, isn't it.

Note that in equation 1.3a, we see “feet” in both the numerator (portion above the line) and the demoninator (portion below the line). This implies that we are both multiplying and dividing by feet. Rather than rewriting equation 3 as shown in equation 1.3a, it is common practice to simply remove both directly. This practice is referred to as “canceling out.” For example, we might say that “the feet cancel out” as we erase them from the equation.

Now something a bit harder. Suppose a distance is 3 miles. How many inches is it? Well, we know that there are 12 inches in a foot, and 5280 feet in a mile. So our equation becomes:

$$1.4 \quad X = (3 \text{ mile}) \left(\frac{5280 \text{ feet}}{\text{mile}} \right) \left(\frac{12 \text{ inch}}{\text{feet}} \right)$$

(Again, we're free to interchange singular and plural forms as we please.) In this example, we're multiplying by one, twice. Our miles cancel out, and our feet cancel out, leaving us with inches. Completing the multiplication gives us the answer:

$$X = 190,080 \text{ inches}$$

In this type of situation, it is important to know what *is* one and what *is not* one. Provided that you know that two quantities are equal, then you can use them to make one: simply divide the first by the second (as noted in equation 1.2). And anything multiplied by one remains the same. As shown above, if we have 3 miles, then we simultaneously have 190,080 inches. The two are the same. And we have arrived at this knowledge simply by knowing what *is* one and by multiplying by one.