Oscillations
(Rotation Without Angles)
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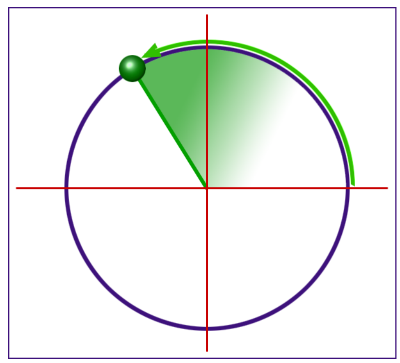
Many, many, many… um many things in nature oscillate. And nearly all oscillations can be represented as sine waves or groups of superimposed sine waves. If the oscillation fades away — that is, if it is damped — then the sine waves are multiplied by damping functions. Because very few oscillations continue indefinitely, damping functions are almost always present. Sine waves and damping functions are everywhere. They can be used to represent a weight on a spring, a piston in a diesel engine, the hands of a clock, the orbiting planets. They truly are everywhere, so if you think that you can get by without them, think again…

You probably noticed the parenthetical "Rotation Without Angles" in the title of this lesson. Certainly this sounds odd. After all, what is a rotation without an angle?

In this illustration, a green ball goes round and round on a wire. With each rotation, the angular motion (displacement) increases by 380º, or 2π radians.



Now suppose you were to view the ball and wire from the left edge of the illustration. What would you see? The ball would simply oscillate up and down, moving quickly in the middle of the stroke and slowly at the top and bottom of the stroke.

In other words, when viewed from the edge, the angular position is no longer apparent and the ball appears simply to move up and down. Hence, an oscillation can be thought of as a "rotation without an angle."

What is the ball's vertical displacement? Very simply,

1.1 $$Y = r * \sin(A)$$

where *A* is the angular displacement and *r* is the circle's radius. If *A* changes very speedily, then the ball will move from top to bottom very quickly. If *r* is large, then the distance between top and bottom will also be large.

It is customary to represent *A* as

1.2 $$A = 2\pi * \omega * t$$

where ω (omega) is the frequency, which is the number of complete cycles per second, and *t* is time in seconds. This means that if $\omega = 4$, then *A* will increase by a full revolution (cycle) every ¼ second.

What About Damping?

Typically, damping is represented using exponential functions. If you have a scientific calculator, exponential functions use the "e^x" button. In Lingo, exponentials use the exp(x) function, which is the same as e^x. If, for example, *x* is 2, then $e^x = e*e$. Similarly, exp(2) is $e*e$.

In the context of damping, it is customary to use -*kt,* rather than *x*, where *k* is the decay constant and *t* is time in seconds.

What is *e*? It is the constant 2.718282 (approximately).

Interestingly, $e^{-0.693} \approx 0.5$

This implies that if k = 0.693, then exp(-kt) shrinks by approximately ½ every second. That is,

Exp(-0.693*1) = 0.5
Exp(-0.693*2) = 0.25
Exp(-0.693*3) = 0.125
Etc.

Note that if the minus sign is omitted, then the progression heads in the other direction, and the resultant *doubles* every second.

To create a damped oscillation, simply multiply the oscillation displacement by the damping factor. That is,

1.3 $$Y = \exp(-kt) * r * \sin(A)$$

If *k* is a small number, then the oscillation will die away slowly. If it is a large number, the oscillation will die very quickly.

And that's all there is to basic oscillations with damping. True, this discussion only scratches the surface of the topic. However, in many, many, many… um many cases, only the simplest basics are necessary.

Wow! This is so easy!